SO(10) GUT MODELS AND COSMOLOGY

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Abstract. $SO(10)$ grand unified models have an intermediate symmetry group in between $SO(10)$ and $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Hence they lead to a prediction for proton lifetime in agreement with the experimental lower limit. This paper reviews the recent work on the tree-level potential and the one-loop effective potential for such models, with application to inflationary cosmology. The open problems are the use of the most general form of tree-level potential for $SO(10)$ models in the reheating stage of the early universe, and the analysis of non-local effects in the semiclassical field equations for such models in Friedmann-Robertson-Walker backgrounds.

1. Introduction

A key tool in modern particle physics is the spontaneous symmetry breaking mechanism, which makes it possible to unify electroweak and strong interactions in such a way that all mass terms for vector mesons are generated without violating gauge invariance and perturbative renormalization. Among the various possible choices for a local symmetry group, models which rely on the $SO(10)$ group are still receiving careful consideration in the literature. The main motivations are as follows [1–3]:

(i) $SO(10)$ models make it possible to obtain masses of the lepto-quarks mediating proton decay which are higher than the ones found within the minimal $SU(5)$ model. Hence one recovers agreement with the experimental lower limit of $9. \times 10^{32}$ years for proton decay.

(ii) All exceptional gauge groups contain $SO(10)$ as a subgroup.

(iii) Exact cancellation of chiral anomalies is obtained.

(iv) The only $SO(2n)$ group which contains a 16-dimensional representation is indeed $SO(10)$. This makes it possible to accommodate fermions and antifermions of each generation in a single 16-dimensional complex representation, while the introduction of exotic or mirror particles is avoided.

(v) On requiring that the GUT group should contain $SU(2)_L$ and $B - \alpha L$ as local symmetries, while no exotic particles or mirror fermions should occur, one finds that $SO(10)$ is unique and has $B - L$ as generator (hence $\alpha = 1$).

(vi) $SO(10)$ models predict masses for τ- and μ-neutrinos of the order of magnitude relevant for cosmology and solar-neutrino astrophysics (including the effects of renormalization-group equations [3]).

Section 2 outlines properties and symmetry-breaking pattern for $SO(10)$ models. Sections 3 and 4 are devoted to the tree-level potential and one-loop effective potential respectively. The one-loop analysis is performed in a cosmological background, i.e. de Sitter space. Results and open problems are presented in section 5.

2. Structure and symmetry breaking of $SO(10)$ models

The group $SO(10)$ consists of all 10×10 orthogonal matrices with unit determinant, and with the usual product rules. It has 45 generators, say T_{ij} $(i, j = 0, 1, ..., 9)$, which obey the commutation relations [4]

$$\left[T_{jk}, T_{lm}\right] = i\left(\delta_{jl}T_{mk} + \delta_{jm}T_{kl} + \delta_{kl}T_{jm} + \delta_{km}T_{lj}\right) . \tag{2.1}$$

Given the vector irreducible representation $\underline{10}$ of $SO(10)$, the action of the generators on $\varphi_l \in \underline{10}$ is given by $T_{jk}\varphi_l \equiv i\left(\delta_{kl}\varphi_j - \delta_{jl}\varphi_k\right)$.

To break the symmetry spontaneously, one needs a Higgs field belonging to one or more irreducible representations of the gauge group $SO(10)$. In particular, one is interested (see section 4) in the most general, renormalizable and conformally invariant Higgs potential constructed by using only the 210-dimensional irreducible representation. Such a representation has four independent quartic invariants, i.e. the fourth power of the norm of the Higgs field and three non-trivial invariants [4]. The tensor product of two $\underline{210}$ representations, jointly with symmetrization, yields the fundamental Clebsch-Gordan decomposition [4]

$$(\underline{210} \otimes \underline{210})_{\text{sym}} = \underline{1} \oplus \underline{45} \oplus \underline{54} \oplus \underline{210} \oplus \underline{770} \oplus (\underline{1050} \oplus \overline{\underline{1050}}) \oplus \underline{4125} \oplus \underline{8910} \oplus \underline{5940} , \tag{2.2}$$

where $\underline{45}$ denotes the 45-dimensional irreducible representation of $SO(10)$, and similarly for the others.

The symmetry breaking process of $SO(10)$ models consists of the following three steps:

$$SO(10) \xrightarrow{M_X} G' \xrightarrow{M_R} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \longrightarrow SU(3)_C \otimes U(1)_Q . \tag{2.3}$$

In the first stage, symmetry is broken at the scale M_X of order 3.2×10^{15} GeV, and one gets an intermediate symmetry group G' which may take the forms $\Omega_1 \times D, \Omega_1, \Omega_2 \times D, \Omega_2$, where $\Omega_1 \equiv SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$, $\Omega_2 \equiv SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ and D is the discrete left-right interchanging symmetry [3, 4]. If $G' = \Omega_1 \times D$, the corresponding Higgs field Φ belongs to the 54-dimensional irreducible representation of $SO(10)$. Otherwise, Φ belongs to the 210-dimensional irreducible representation. The second symmetry breaking occurs at the scale M_R of order 10^{11} GeV, by means of a scalar field $\varphi_+ \in \underline{126}$, and a scalar field $\varphi_- \in \overline{\underline{126}}$. Last, the symmetry breaking to $SU(3)_C \otimes U(1)_Q$ involves a scalar field $\rho \in \underline{10}$.

3. Tree-level potential

Following Ref. [3], we are here interested in the orbit structure of the tree-level potential for $SO(10)$ GUT models in flat space-time. For this purpose, we denote by $\Phi, \varphi_+, \varphi_-$ and ρ the generic elements of the $\underline{210}, \underline{126}, \underline{\overline{126}}$ and $\underline{10}$ representations, respectively. Hence we can write the tree-level potential in the form [3]

$$V(\Phi, \varphi_+, \varphi_-, \rho) = V_0 + V_\Phi(\Phi) + V_\varphi(\varphi_+, \varphi_-) + V_{\Phi,\varphi}(\Phi, \varphi_+, \varphi_-) + V_{\Phi,\varphi,\rho}(\Phi, \varphi_+, \varphi_-, \rho) . \quad (3.1)$$

The term V_0 only depends on the fields' norms and hence cannot affect the direction of the potential minimum [5]. The first symmetry breaking is instead realized by $V_\Phi(\Phi)$, which breaks $SO(10)$ to the group G' of section 2. The second symmetry breaking is then realized by means of $\left[V_\varphi(\varphi_+, \varphi_-) + V_{\Phi,\varphi}(\Phi, \varphi_+, \varphi_-)\right]$, from G' down to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Last, the term $V_{\Phi,\varphi,\rho}(\Phi, \varphi_+, \varphi_-, \rho)$ makes it possible to break the symmetry down to $SU(3)_C \otimes U(1)_Q$. The detailed formulae for the various contributions to (3.1) are a bit lengthy and can be found in Ref. [3]. However, we can describe a few relevant points. On denoting by $(\Phi\Phi)_d$ the d-dimensional representation in the tensor product $210 \otimes 210$, V_Φ is a linear combination of the norms of $(\Phi\Phi)_{45}, (\Phi\Phi)_{54}, (\Phi\Phi)_{210}$ and of the singlet for $(\Phi\Phi)_{210} \times \Phi$. With a similar notation for $(\varphi\varphi), (\Phi\varphi)$ and $(\rho\rho)$ terms, one finds that $V_\varphi(\varphi_+, \varphi_-)$ is obtained from the norms of $(\varphi_+\varphi_+)_{4125}, (\varphi_-\varphi_-)_{4125}, (\varphi_+\varphi_+)_{1050}, (\varphi_+\varphi_+)_{54}$ and $(\varphi_-\varphi_-)_{54}$. The contribution $V_{\Phi,\varphi}(\Phi, \varphi_+, \varphi_-)$ involves many cross-terms. However, on choosing the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ direction, it only involves $(\Phi\varphi_+)_{126}, \varphi_+$, and the singlet of $(\varphi_+\varphi_-)_{45} \times (\Phi\Phi)_{45}$. Last, denoting by P_{10} the most general third-order polynomial which transforms as a 10-dimensional irreducible representation, the contribution $V_{\Phi,\varphi,\rho}(\Phi, \varphi_+, \varphi_-, \rho)$ involves the singlet of $P_{10} \times \rho$ and the singlet of the tensor product

$$\left(q_1(\Phi\Phi)_{54} + q_2(\varphi_+\varphi_+)_{54} + q_2^*(\varphi_-\varphi_-)_{54}\right) \otimes (\rho\rho)_{54} .$$

4. One-loop effective potential in de Sitter space

Within the framework of inflationary cosmology, the quantization of non-Abelian gauge fields has been recently studied in the case of SU(5) GUT theories [6–8]. In this case one starts from a bare, Euclidean-time Lagrangian

$$L = \frac{1}{4}\text{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}) + \frac{1}{2}\text{Tr}(D_\mu\varphi)(D^\mu\varphi) + V_0(\varphi) , \quad (4.1)$$

where both the gauge potential, A^μ, and the Higgs scalar field, φ, are in the adjoint representation of $SU(5)$. The background four-geometry is de Sitter space with S^4 topology. The background-field method is then used, jointly with the gauge-averaging term first proposed by 't Hooft

$$L_{\text{g.a.}} = \frac{1}{2}\widetilde{\alpha} \, \text{Tr}\left(\nabla_\mu A^\mu - i\mathcal{G} \, \widetilde{\alpha}^{-1}[\varphi_0, \varphi]\right)^2 . \quad (4.2)$$

This particular choice is necessary to eliminate from the total action cross-terms involving $\text{Tr}(\nabla_\mu A^\mu)$ and the commutator $[\varphi_0, \varphi]$, where φ_0 is a constant background Higgs field. We now bear in mind that, by virtue of the Coleman-Weinberg mechanism [9], only gauge-field loop diagrams contribute to the symmetry-breaking pattern in the early universe. Thus, after sending $\tilde{\alpha} \to \infty$ (Landau condition), and denoting by $\Omega = \frac{8}{3}\pi^2 a^4$ the volume of a four-sphere of radius a, the resulting one-loop effective potential is [6]

$$V^{(1)}(\varphi_0) \sim V_0(\varphi_0) + \frac{1}{2\Omega}\log \det \mu^{-2}\Big[\delta_{ab}(-g_{\sigma\tau}\Box + R_{\sigma\tau}) + g_{\sigma\tau}M^2_{ab}(\varphi_0)\Big] , \qquad (4.3)$$

since the ghost determinant cancels the longitudinal one. Denoting by ψ the logarithmic derivative of the Γ function, and defining the functions $\mathcal{A}$ and P by means of

$$\mathcal{A}(z) \equiv \frac{z^2}{4} + \frac{z}{3} - \left[\int_2^{\frac{3}{2}+\sqrt{\frac{1}{4}-z}} + \int_1^{\frac{3}{2}-\sqrt{\frac{1}{4}-z}}\right] y\Big(y - \frac{3}{2}\Big)(y-3)\psi(y)dy , \qquad (4.4)$$

$$P(z) \equiv \frac{z^2}{4} + z , \qquad (4.5)$$

one thus finds for the $SU(5)$ model [6]

$$V^{(1)}(\varphi_0) \sim V_0(\varphi_0) - \frac{1}{2\Omega}\sum_{l=1}^{24}\Big[\mathcal{A}(a^2 m_l^2) + P(a^2 m_l^2)\log(\mu^2 a^2)\Big] , \qquad (4.6)$$

where the m_l^2 are the 24 eigenvalues of the mass matrix M^2_{ab}.

In the case of $SO(10)$ GUT models, the same method used for $SU(5)$ shows that the one-loop effective potential, $V^{(1)}$, takes the form

$$V^{(1)} \sim \widehat{V}_c - \frac{1}{2\Omega}\sum_{i=1}^{45}\Big[\mathcal{A}(a^2 m_i^2) + P(a^2 m_i^2)\log(\mu^2 a^2)\Big] , \qquad (4.7)$$

where, with the notation of Ref. [4], the renormalizable and conformally invariant Higgs potential constructed by using only the 210-dimensional irreducible representation is

$$\widehat{V}_c = \Big(\frac{\alpha}{8}f_\alpha + \frac{\gamma}{4}f_\gamma + \frac{\delta}{9}f_\delta + (\lambda - \delta)\Big)\|\phi_0\|^4 + \frac{R}{12}\|\phi_0\|^2 , \qquad (4.8)$$

where R denotes the scalar curvature of the background four-metric. Note that, while (4.7) holds for any irreducible representation of $SO(10)$, we are only able to evaluate a *particular* form of the one-loop effective potential, once the $SU(3) \otimes SU(2) \otimes U(1)$ invariance for the mass matrix is required to agree with electroweak symmetry [4]. On denoting by (l, r, x) the tensor which behaves as an l-dimensional representation under $SU(3)$, r-dimensional under $SU(2)$, and takes a value x when acted upon by the $U(1)$ generator, the non-vanishing eigenvalues of the mass matrix occurring in (4.7) are then found to be $m^2_{(1,1,1)} = m^2_{(1,1,-1)}$ with degeneracy 1, $m^2_{(3,1,2/3)}$ with degeneracy 3, $m^2_{(3,2,1/6)}$ with degeneracy 6, and $m^2_{(3,2,-5/6)}$ also with degeneracy 6 [4].

5. Results and open problems

The work in Refs. [7, 8] led to a deeper understanding of the symmetry-breaking pattern for $SU(5)$ models in de Sitter space first found in Ref. [6], on combining analytic and numerical techniques with the group-theoretical methods used in Ref. [5]. Although curvature effects modify the flat-space effective potential by means of the complicated special function defined in (4.4), the inflationary universe can only slide into either the $SU(3) \otimes SU(2) \otimes U(1)$ or $SU(4) \otimes U(1)$ extremum [6, 7].

Attention was then focused on $SO(10)$ GUT models for the reasons described in section 1. On using the particular one-loop effective potential of section 4, with the mass matrix relevant for the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry-breaking direction, it was found in Ref. [4] that, as far as the absolute-minimum direction is concerned, the flat-space limit of the one-loop calculation with a de Sitter background does not change the results relying on the tree-level potential in flat space-time. Moreover, even when curvature effects are no longer negligible in the one-loop potential, it was found that the early universe can only reach the $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ minimum [4].

However, a constant Higgs field, with de Sitter four-space as a background in the corresponding one-loop effective potential, is only a mathematical idealization. A more realistic description of the early universe is instead obtained on considering a dynamical space-time such as the one occurring in Friedmann-Robertson-Walker models. As a first step in this programme, electrodynamics for self-interacting scalar fields in spatially flat Friedmann-Robertson-Walker space-times was studied in Ref. [10]. In the case of exponentially expanding universes, the equations for the Bogoliubov coefficients describing the coupling of the scalar field to gravity were solved numerically. They yield a non-local correction to the Coleman-Weinberg effective potential which does not modify the pattern of minima found in static de Sitter space. However, such a correction contains a dissipative term which, accounting for the decay of the classical configuration in scalar field quanta, may be relevant for the reheating stage [11–13]. The physical meaning of the non-local term in the semiclassical field equation was investigated by evaluating its contribution for various background-field configurations [10].

More recently, the work in Ref. [11] has studied the flat-space limit of the one-loop effective potential for $SO(10)$ GUT models in spatially flat Friedmann-Robertson-Walker cosmologies. The numerical integration of the corresponding field equations shows that a sufficiently long inflationary stage is obtained for suitable choices of the initial conditions. However, a large e-fold number is only achieved by means of a severe fine tuning of these initial conditions. Moreover, in the direction with residual symmetry $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$, one eventually finds parametric resonance [12, 13] for suitable choices of the free parameters of the tree-level potential. This leads in turn to the end of inflation [11].

At least two exciting problems are now in sight:

(i) Can one use the complete form of the tree-level potential outlined in section 3 to get a better understanding of the reheating stage in the early universe ? Indeed, the various cross-terms occurring in (3.1) are negligible during the inflationary era, but they are important

when the second stage of reheating [12, 13] is considered, since they can mediate the decay of the massive Higgs into lighter particles [11, 14].

(ii) How to extend the analysis of non-local effects performed in Ref. [10] to $SO(10)$ GUT models in Friedmann-Robertson-Walker cosmologies ? Can such an investigation improve the current understanding of dissipative and non-dissipative effects [10, 15] in the early universe ?

Although our presentation is far from being exhaustive, the results and open problems seem to add evidence in favour of a new age being in sight in cosmology, particle physics and quantum field theory in curved space-time. We feel that substantial progress can only result from a fertile interplay between particle-physics phenomenology and the powerful methods of relativistic cosmology and quantum field theory. We leave the reader with this thought, and we hope he will share our excitement in the course of studying the fundamental problems of the physics of the early universe.

Acknowledgments

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